

1. The polar coordinates of a point are given. Find the rectangular coordinates of each point.

$$x = r \cos \theta, y = r \sin \theta$$

a) $(5, 300^\circ) \left(\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$ c) $(-3, 296^\circ) (-1.315, 2.696)$

b) $(2, -\frac{3\pi}{4}) (-\sqrt{2}, -\sqrt{2})$ a.) $x = 5 \cos 300^\circ = 5 \cdot (\frac{1}{2}) = 5/2$
 $y = 5 \sin 300^\circ = 5 \cdot (-\sqrt{3}/2) = -5\sqrt{3}/2$

b.) $x = 2 \cos(-\frac{3\pi}{4}) = 2 \cdot (-\frac{\sqrt{2}}{2}) = -\sqrt{2}$ c.) $x = -3 \cos(296^\circ) = -1.315$
 $y = 2 \sin(-\frac{3\pi}{4}) = 2 \cdot (-\frac{\sqrt{2}}{2}) = -\sqrt{2}$ $y = -3 \sin(296^\circ) = 2.696$

2) The rectangular coordinates of a point are given. Find polar coordinates for each point.

$$x^2 + y^2 = r^2 \rightarrow r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(8.3)^2 + (4.2)^2} = 9.302$$

$$\theta = \tan^{-1}\left(\frac{4.2}{8.3}\right) = 26.841^\circ$$

Q 3 a) $(-2, -2\sqrt{3}) \left(4, \frac{4\pi}{3}\right)$

Q 1 c) $(8.3, 4.2) (9.302, 26.841^\circ)$

a.) $r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4$

$\tan \theta = \frac{y}{x} \rightarrow \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

* ADD π to the angle to obtain A coordinate
 IN Q3 $\rightarrow \frac{\pi}{3} + \pi = \frac{4\pi}{3}$

Q 2 b) $(-5, 5) (5\sqrt{2}, \frac{3\pi}{4})$

$r = \sqrt{(-5)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

3) Transforming an Equation from Polar to Rectangular Form:

Write the equation $r^2 + 4r \sin \theta - 8r \cos \theta = 5$ as an equation in rectangular coordinates

(x, y) .

$$x^2 + y^2 + 4y - 8x = 5$$

$$(x^2 - 8x + 16) + (y^2 + 4y + 4) = 5 + 16 + 4$$

STANDARD Form of an equation of a circle

$$\rightarrow (x-4)^2 + (y+2)^2 = 25 \rightarrow \boxed{(4, -2)}$$

* circle w/ a center at $(4, -2)$ & a radius = 5

4) Transforming an Equation from Rectangular to Polar Form:

Write the equation $4x^2y = 1$ using polar coordinates (r, θ) .

$$4(r \cos \theta)^2(r \sin \theta) = 1$$

$$(4r^2 \cos^2 \theta)(r \sin \theta) = 1$$

$$\frac{4r^3 \cos^2 \theta \sin \theta}{4} = \frac{1}{4}$$

$$\boxed{r^3 \cos^2 \theta \sin \theta = \frac{1}{4}}$$

* must stay in domain of \tan^{-1} when solving
 * ADD π to $-\frac{\pi}{4}$ to get into Q2